EST II – Individual Subject Test

Level 1

Student's Name _____

National ID _____

Test Center: _____

Subject: Math

Duration: 60 minutes

50 Multiple Choice Questions

Instructions:

- Place your answer on the answer sheet. Mark only one answer for each of the multiple choice questions.

- Avoid guessing. Your answers should reflect your overall understanding of the subject matter.

- Calculators are allowed. When a calculator is used, be aware of switching between radian mode and median mode.

- A formula sheet is available at the end of the booklet for your reference.

1. A plane intersects a sphere of radius 4cm in a circle. Which of the following cannot be the area of this circle?

   A. 4π cm^2
   B. 8π cm^2
   C. 12π cm^2
   D. 16π cm^2
   E. 24π cm^2

2. If $3^n \times 9^m \times 27^p = 1$, then which of the following is the correct expression of n in terms of m and p?

   A. $n = \dfrac{1}{mp}$
   B. $n = -3m - 2p$
   C. $n = 1 - 2m - 3p$
   D. $n = -m - p$
   E. $n = -2m - 3p$

3. The three lines l, d, and t in the plane are such that they are parallel and d is equidistant from l and t. If the equation of l is $y = 2x + 70$ and the equation of d is $y = 2x + 10$, which of the following is the equation of line t?

   A. $2x - y = -40$
   B. $2x - y = 50$
   C. $2x - y = 40$
   D. $x - 2y = 50$
   E. $x - 2y = -40$

4. If a and b are real numbers, $i^2 = -1$, and $a + bi = (2 + 3i)^2$, then which of the following is the average of a and b?

   A. 2.5
   B. 3
   C. 3.5
   D. 5
   E. 12.5

5. The volume of a certain pyramid of rectangular base is 12. We alter its dimensions in such a way that the volume does not change. If the height has been doubled, which of the following could be possible alterations to the length L and the width W of the base of the pyramid?

   A. Multiply L by 12 and divide W by 6
   B. Multiply L by 1 and divide W by 0.5
   C. Multiply L by 9 and divide W by 18
   D. Divide both L and W by 0.5
   E. Multiply both L and W with 0.5

2

6. If f is the function defined by $f(x) = \dfrac{3x^2-4x+1}{2x^2+7x-9}$, then which of the following is a root of $f(x)$?

I. $-\dfrac{9}{2}$

II. $\dfrac{1}{3}$

III. 1

A. Only I
B. Only II
C. Only III
D. II and III
E. I, II, and III

7. In the adjacent figure, B is a point on the circle of center A and diameter DC such that BC = 20 and BD = 15. What is the perimeter of triangle BDC?
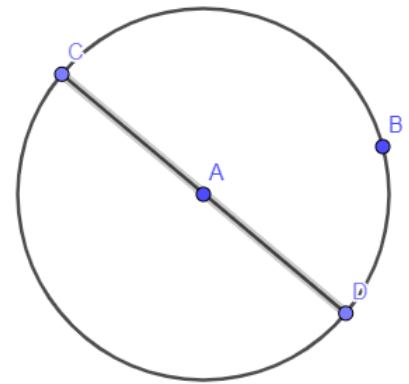
A. 17.5
B. 35
C. 60
D. 150
E. 60π

8. If $0° \leq \theta < 90°$, then $\dfrac{1}{1-sin\theta} =$

A. $\dfrac{1}{cos^2\theta} + \dfrac{tan\theta}{cos\theta}$

B. $1 - \dfrac{1}{sin\theta}$

C. $1 + sin\theta$

D. $tan^2\theta + \dfrac{tan\theta}{cos\theta}$

E. $\dfrac{1}{cos^2\theta} + sin^2\theta$

9. When fidget spinners were first introduced in the market, Leonard decided to sell them in his toy shop. The number of spinners he sold every week is modeled by $N(w) = -w^2 + 10w$, where w is the number of weeks starting the first week of February and $N(w)$ is the number of spinners sold each week in hundreds. One particular week, Leonard realized that his sales were greater than the sales in any other week. What is this maximum number of sales?

A. -2500
B. 500
C. 1000
D. 2500
E. 5000

3

10. Celine and Lia are standing on a horizontal track at points A and D respectively. Celine is 1.5m tall and Lia is 1.2m tall (that is AC = 1.5m and DL = 1.2m). Celine's shadow occupies the length AB and Lia's shadow occupies the length BD on the ground so the two shadows meet at their heads. The angles between the ground and the ray from the top of the head of each individual to the touching point of the shadows is shown. How far apart are Celine and Lia standing?
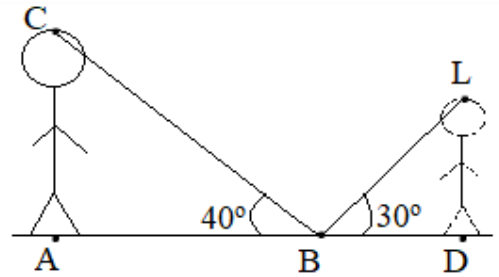
FIGURE IS NOT DRAWN TO SCALE

A. 1.788 m
B. 2.078 m
C. 2.7 m
D. 3.344 m
E. 3.866 m

11. Consider the function f defined by $f(x) = (m - 1)x^3 - 3m - 1$, where m is a real number. If the graph of f has a y−intercept of 2, then $f(1) =$

A. −6.4
B. −1
C. 0
D. 1
E. 2.2

12. If y varies directly with z and inversely with v, then which of the following could be a correct expression of z as a function of y and v?

A. $z = 3yv$
B. $z = \frac{1}{6}\frac{y}{v}$
C. $z = \frac{7}{6}\frac{v}{y}$
D. $z = 8\frac{y}{v}$
E. $z = \frac{1}{6}(y + v)$

13. If $\sqrt{a} + \sqrt{b} = 4.6$ and $a + b = 13.46$, then $ab =$

A. 1.21
B. 3.85
C. 7.7
D. 12.25
E. 14.8225

4

14. In a certain country, the ratio of male individuals who got infected by the COVID-19 virus to the female individuals who got infected by the same virus is 6 to 4. Moreover, it was observed that only 9% of the males and 5% of the females who get infected by the virus got severe symptoms. If the number of COVID-19 cases in the country is 1200, how many females have severe symptoms?

    A. 24
    B. 162
    C. 600
    D. 1200
    E. 1800

15. Which of the following is equal to $\dfrac{5^{202}+5^{200}}{25^{100}}$?

    A. 25
    B. 26
    C. 2^{202}
    D. 5^{202}
    E. 10^{202}

16. Each day, Jack's donut shop starts with 100 donuts. Throughout the day, some donuts are sold, some are discarded, and some more is made according to the following principle: for every three donuts sold, one more is made. If on a particular day, Jack sold m donuts and discarded d donuts, and at the end of the day had no donuts left, which of the following gives the correct relation between m and d?

    A. $2m + 3d = 300$
    B. $3m + 2d = 300$
    C. $3d - 2m = 300$
    D. $2d - 3m = 100$
    E. $m + d = 100$

17. Consider the set $\{-3, 0, 7, a, b, 20\}$ whose elements are arranged in increasing order, and a and b are real numbers. If the median of this set above is 12, which of the following is a possible value of b?

    I. 8
    II. 17
    III. 20

    A. Only I
    B. Only II
    C. Only III
    D. I and II
    E. II and III

18. Let f be the function defined by $f(x) = x^4 - 3x^2 - k$ where k is a real number. For which values of k does the line d of equation $y = 1$ cut the graph of f in 4 distinct points?

A. -3.7
B. -1.9
C. -1
D. 0
E. 1.2

19. The product of three consecutive integers is equal to the middle integer. What is the smallest of these integers?

A. $-1 - \sqrt{2}$
B. $1 + \sqrt{2}$
C. -1
D. 0
E. 1

20. A line in the plane passes through the 3 distinct points (a, b), (c, d), and (e, f). Then all of the following give a correct relation between $a, b, c, d, e,$ and f except:

A. $\dfrac{c-a}{d-b} = \dfrac{c-e}{d-f}$
B. $\dfrac{d-b}{d-f} \times \dfrac{c-e}{a-c} = 1$
C. $\dfrac{d-b}{d-f} \times \dfrac{c-e}{c-a} = 1$
D. $\dfrac{c-a}{d-b} + \dfrac{c-e}{f-d} = 0$
E. $\dfrac{d-b}{d-f} - \dfrac{c-a}{c-e} = 0$

21. Consider the parabola of equation $y = x^2 + 22x - 320$. What are the coordinates of the midpoint of the segment joining the two x-intercepts of the parabola?

A. $(-11, 0)$
B. $(0, -11)$
C. $(11, 0)$
D. $(0, 11)$
E. $(0, -160)$

22. In a certain company of 400 employees, 70% wear face masks at all times. Out of those who do not wear face masks, 15% got the disease. Out of those who do wear a mask, only 5% got the disease. If an employee is chosen at random from this company, what is the probability that he or she got the disease?
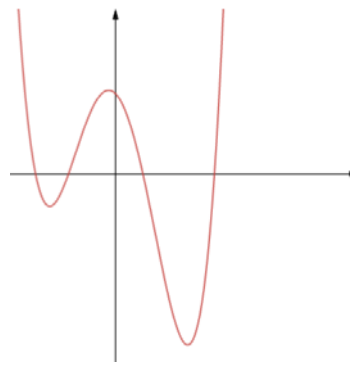
A. 0.35
B. 0.08
C. 0.09
D. 0.12
E. 0.35

23. The adjacent graph is that of a function $f(x)$. The graph of the function g is obtained by translating the graph of f, that is $g(x) = f(x + a) + b$ for some real numbers a and b. Which of the graphs below cannot be that of $g(x)$ for any real values of a and b?
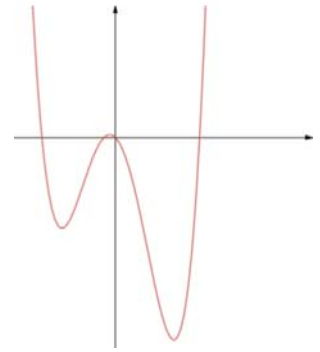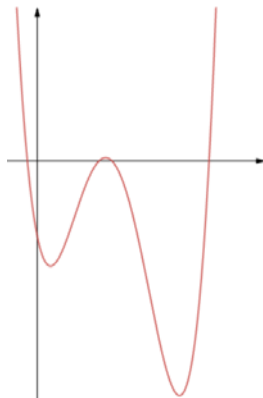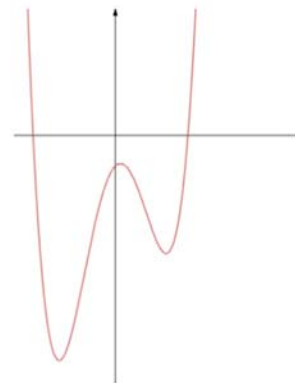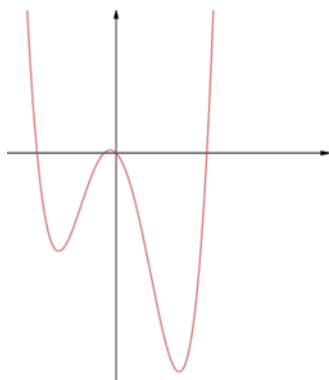
A.
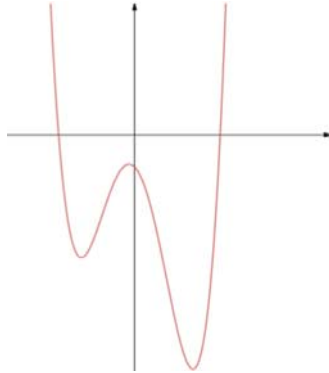
B.

C.

D.

E.

24. If f is the function defined by $f(x) = (2x + 1)^3$, for which of the following values of x is $f(f(x)) = f(x)$?

A. -2
B. -1
C. 0
D. 1
E. 2

25. The range of the function defined by $f(x) = |1 - x^2| + 2$ is:

A. $f(x) > 2$
B. $f(x) > 3$
C. $f(x) < 2$
D. $f(x) \geq 2$
E. $f(x) \leq 2$

26. A circle inscribed in a square has an area of 36π. What is the length of the diagonal of the square?

A. 8.485
B. 12
C. 16.97
D. 36
E. 50.912

27. If $f(x) = x^2 - 4x + 1$, which of the following is also equal to $f(x)$?

A. $f(x - 4)$
B. $f(2 - x)$
C. $f(4 - x)$
D. $f(4 + x)$
E. $f(4) - f(x)$

28. C is a circle tangent to the two straight lines of equations $y = 2$ and $y = -4$. Which of the following could be the equation of the circle?

A. $(x + 1)^2 + y^2 = 9$
B. $(x - 1)^2 + (y + 1)^2 = 3$
C. $(x + 4)^2 + (y + 2)^2 = 9$
D. $(x + 1)^2 + y^2 = 3$
E. $(x + 2)^2 + (y + 1)^2 = 9$

29. Consider all spheres of nonzero volume having as their center the point A in space. How many of these spheres can have a surface area whose value is numerically equal to the volume of the sphere?

A. None
B. 1
C. 2
D. 3
E. Further information is required to determine the answer

30. The two lines m and n of respective equations $3x - y = 1$ and $ax + by = 2$ are parallel, where $a < 0$ and b is a real number. Which of the following may be true about b?

I. $b = -2.31$
II. $b = 1.04$
III. $b = 0$

A. Only I
B. Only II
C. I and III
D. II and III
E. I, II, and III

31. In the adjacent figure, ABCD is a rectangle such that BC = 4 and AB = 3, AEC is a triangle such that the two angles $\angle AEC$ and $\angle ACE$ are equal. What is the perimeter of triangle AEC?
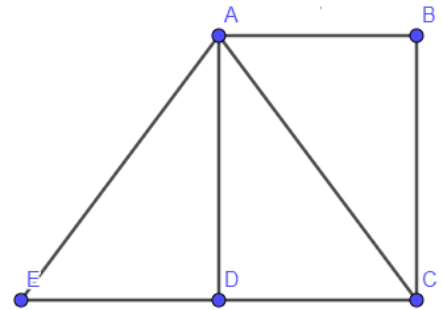
A. 10
B. 12
C. 14
D. 16
E. 18

32. A, C, E, and D are four collinear points in this order such that $AC = a, ED = b,$ and $AE + CD = 12$. Which of the following is a correct expression of CE in terms of a and b.

A. $CE = 6 - \frac{a+b}{2}$
B. $CE = \frac{a+b}{2}$
C. $CE = 12 - (a + b)$
D. $CE = 6 - (a + b)$
E. $CE = 12 - \frac{a+b}{2}$

33. The triangle shown in the adjacent figure (not drawn to scale) is right angled at G with GF = 4 and $\angle GFH = 60°$. I is the midpoint of segment FH an GJ is the altitude drawn from G to FH. What is the area of triangle GJI?
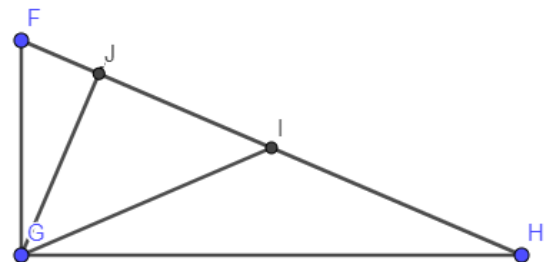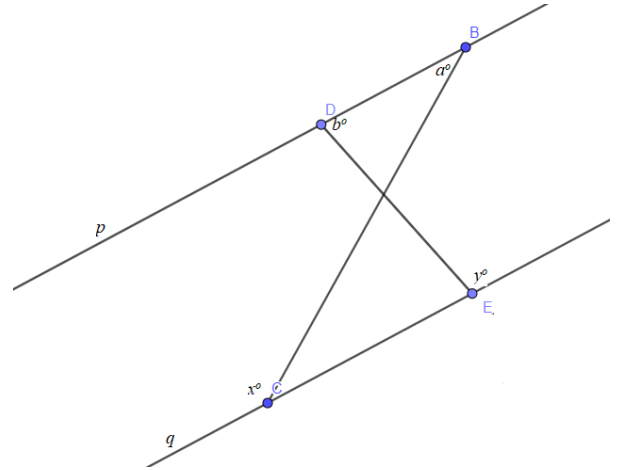
A. 3.46
B. 4.62
C. 6
D. 6.93
E. 13.86

34. In the adjacent figure, p and q are two parallel lines. B and D are points on p; C and E are points on q. Which of the following statements is true?

I. $a^o + b^o = x^o + y^o$
II. $a^o, b^o, x^o,$ and y^o can be the values of the four angles of a quadrilateral.
III. $a^o + x^o = b^o + y^o$

A. Only I
B. Only II
C. Only III
D. I and II
E. II and III

**35.** If $3k - \dfrac{k}{k+1} = \dfrac{16}{k+1}$, what is a possible value of $k + 1$?

   A. $-\dfrac{8}{3}$
   **B.** $-\dfrac{5}{3}$
   C. 2
   D. 5
   E. 6

36. In the adjacent figure (not drawn to scale), segments AD and AH are perpendicular. The following degree measures are known in terms of b: $\angle ADC = b$, $\angle EGF = 2b + 30$, and $\angle EFG = 150 - 3b$. Which of the following conclusions must be true?

   A. The three triangles ADC, CHE, and EFG are right angled triangles.
   B. Triangle HCE is right angled at H.
   C. Triangles ADC and EFG are similar.
   D. Triangles ADC and HCD are similar.
   E. Only triangle ADC is right angled.
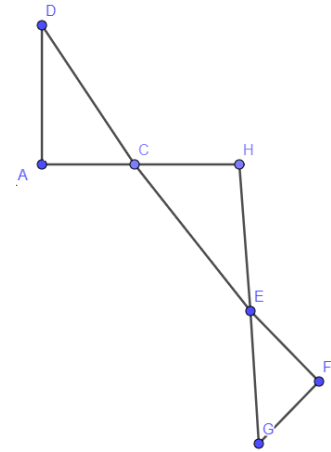
37. In the adjacent figure, lines AC and DE are parallel. The lengths of segments CE, BE, BD, and AD are given in terms of s and z. Which of the following is a correct expression of s in terms of z?

   A. $s = \dfrac{2z}{z-3}$
   B. $s = 2 - \dfrac{3}{2z}$
   C. $s = \dfrac{z+3}{3-2z}$
   D. $s = \dfrac{3(z+1)}{z+3}$
   E. $s = \dfrac{z+3}{z} + 1$
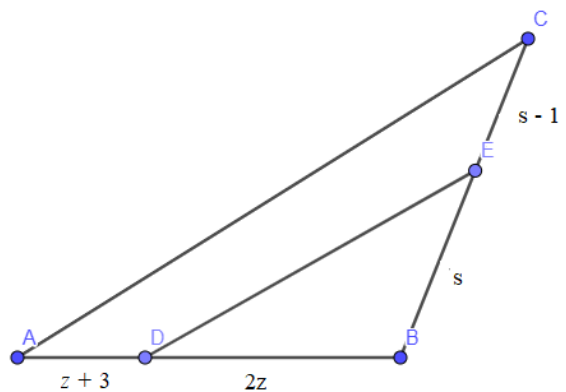
38. The perimeter of a regular hexagon is 24 and the perimeter of a regular octagon is 5. What is the ratio of the length of one side of the hexagon to the length of one side of the octagon?

   A. 0.15625
   B. 0.2083
   C. 1.2
   D. 2.75
   E. 6.4

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39. The equation $\dfrac{1+|x-1|}{|x-1|} = 5$ has two solutions. What is the sum of these two solutions?

A. 0.75
B. 1.25
C. 2
D. 2.5
E. 3.25

40. Let f be the function defined by $f(x) = mx^3 + px^2 + qx - 1$, where $m, p,$ and q are real numbers. If the graph of f passes through the point $(1, 7)$, what is the average of $m, p,$ and q?

A. $\dfrac{7}{3}$
B. 2
C. $\dfrac{8}{3}$
D. 3.5
E. 4

41. If $a(b-1) = 1$ and $12ab = 4$, what is the value of b?

A. -0.667
B. -0.5
C. 0.5
D. 0.667
E. 2

42. If m and n are the roots of the equation $ax^2 + bx + c = 0$, where $a, b,$ and c are real numbers, which of the following quantities does not change when we vary a ?

I. $m.n$
II. $m + n$
III. $\dfrac{1}{m} + \dfrac{1}{n}$

A. Only I
B. Only II
C. Only III
D. I and II
E. I, II, and III

43. If $f(x) = 1.6^x$ and $g(x) = 1.4^x$ and $f(g(0)) = A \times g(f(0))$, what is the value of A?

A. $\dfrac{7}{8}$
B. 1
C. $\dfrac{8}{7}$
D. $\dfrac{25}{56}$
E. $\dfrac{56}{25}$

44. Circles C, P, and Q have radii 1, 2, and 3 respectively. If the three circles are tangent to each other, what is the nature of the triangle formed by joining the three centers of the circles?

A. Isosceles
B. Right angled
C. Right isosceles
D. Equilateral
**E.** Cannot be determined with the information given

45.

Days d	1	2	3	4	5	6	7	8
Cases N	100	350	250	600	723	702	750	790

The table above shows the number of cases of individuals infected with the COVID-19 virus in a certain country every day after Sunday October 18 (so $d = 1$ is Monday October 19, and so on). The government uses a linear regression model and decides to impose total lockdown when the number of cases reaches 1000. Using the linear model, what is the predicted date at which the lockdown will be imposed?

A. October 20
B. October 23
C. October 29
D. October 31
E. November 1

46.

Sales in Millions
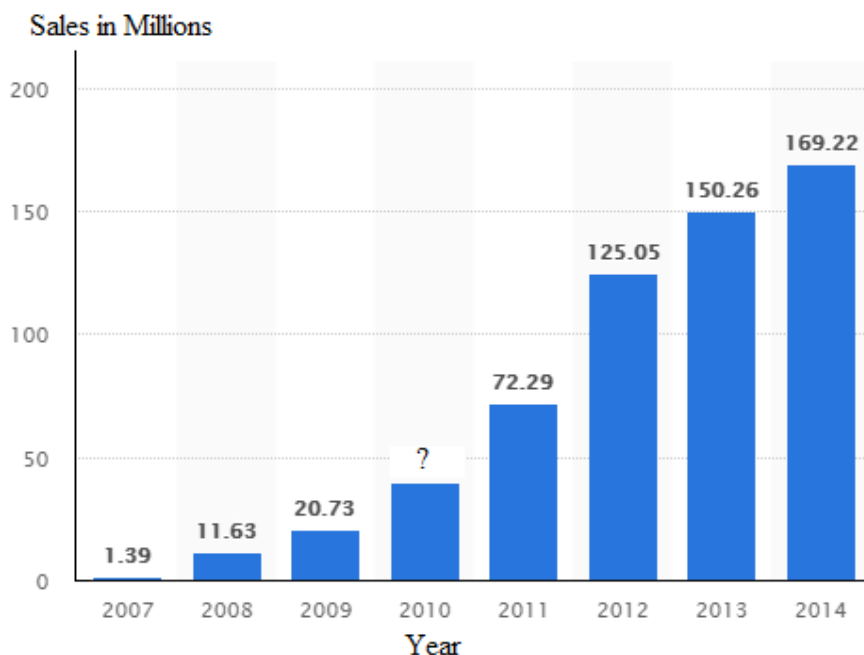
The bar graph given above shows the number of smart phones sold in millions from 2007 when it was first introduced into the market till 2014 except for the year 2010. Knowing that the percent increase from the year 2010 to 2011 for the number of smart phones sold is four times the percent increase from the year 2013 to 2014, what is the approximate number of smart phones sold, in millions, in the year 2010?

A. 14.458
B. 32.35
C. 40.161
D. 25.73
E. 71.52

47. For how many strictly positive integer values of x is the following inequality satisfied?

$$2(3 - x) \geq -14$$

A. 0
B. 1
C. 9
D. 10
E. 11

48. If $f(x) = x^2 + 1$ and $d(x) = \frac{f(x) - f(2)}{x - 2}$, then which of the following is true?

A. $d(x) = f(x)$ for all values of x
B. $d(x) = x - 2$ for all values of x
C. $d(x) = x + 1$ for all values of x except 2
D. $d(2) = f(2)$
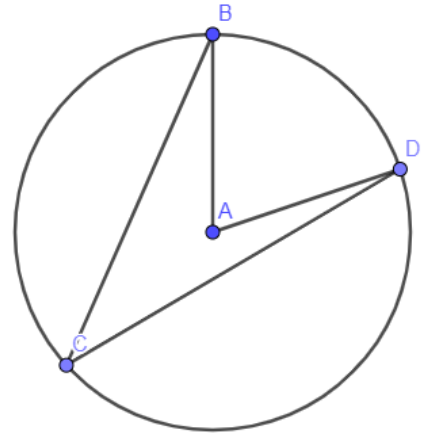E. $d(x) = x + 2$ for all values of x except 2

49. $\begin{cases} 12x - 15y = 17 \\ x + 8y = 12 \end{cases}$

What is the value of $46y - 22x$?

A. -10
B. -5
C. 0
D. 5
E. 10

50. In the adjacent figure, the measure of angle $\angle BAD$ is $80°$. Which of the following could be the measure of angle $\angle BCA$?

A. $38.96°$
B. $40°$
C. $45.02°$
D. $60.13°$
E. $90°$

THE FORMULAS BELOW MAY BE USEFUL IN ANSWERING QUESTIONS ON THIS TEST.

$S = 4\pi r^2$ is the formula for the surface area of a sphere with a radius of r.

$V = \frac{1}{3}\pi r^2 h$ is the formula for a right circular cone with a radius of r and a height of h.

$V = \frac{4}{3}\pi r^3$ is the formula for a sphere with a radius of r.

$V = \frac{1}{3}Bh$ is the formula for a pyramid with a base area of B and a height of h.

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