EST II – Individual Subject Test

Level 1

Student's Name _____

National ID _____

Test Center: _____

Subject: Math

Duration: 60 minutes

50 Multiple Choice Questions

Instructions:

- Place your answer on the answer sheet. Mark only one answer for each of the multiple choice questions.
- Avoid guessing. Your answers should reflect your overall understanding of the subject matter.
- Calculator is allowed. When a calculator is used, be aware of switching between radian mode and median mode.
- Formula sheet is available at the end of the booklet for your reference.

1. How many terms are there in the sequence 4, 8, 12, … 2020?

A. 505
B. 504
C. 510
D. 503
E. 700

2. The sum of two 2-digit numbers is a third 2-digit number. What is the maximum value of the product of these three numbers?

A. 356,000
B. 238,392
C. 970,299
D. 239,580
E. 242,550

3. (U_n) is a sequence such that $U_0 = 2$ and $U_{n+1} = (U_n - 3)^2$. What is the value of U_2?

A. -4
B. 4
C. 2
D. 0
E. 6

4. The registration plate of a car in a country begins with two letters, from the 26 letters in the English Alphabet, followed by four even digits. Neither the letters nor the digits can be repeated. In a certain city in this country, the first letter of every registration plate must be "A", while the last digits cannot be zero. How many possible different registration plates can exist in this city?

A. 1,500
B. 2,400
C. 2,496
D. 13,000
E. 113,400

5. Consider the two matrices $A_{4\times6}$ and $B_{2\times m}$. What should be m so that $B \cdot A$ defined?

A. 2
B. 4
C. 6
D. 5
E. 4×2

6. On a science exam of 40 multiple choice questions, Elvis got 80% of the 15 biology questions correct, 60% of the 15 chemistry questions wrong and 20% of the 10 physics questions wrong. Knowing that Elvis answered all the questions, what percentage of all the questions did he get wrong?

A. 35%
B. 65%
C. 50%
D. 72.5%
E. 54%

7. What is the value of x in the equality $2ix(i-1) = (i-1)^2 - 2$ knowing that $i = \sqrt{(-1)}$?

A. -2
B. -1
C. 0
D. 2
E. 1

8. If $|x-2| < 4$, then $|x-3|$ is less than

A. 4
B. 2
C. 3
D. 5
E. 0

9. A bicycle store finds that N, the number of bikes sold, is related to d, the number of dollars spent on advertising by the relation $N = 51 + 10ln\left(\frac{d}{10} + 2\right)$.

If the average profit is \$35 per bike, is it worthwhile to spend \$1000 on advertising?

A. No, because the profit will be less than the amount spent on advertising.
B. No, because the profit will be \$97.
C. Yes, because advertisement is important.
D. Yes, because the profit is bigger than \$1000.
E. No, because the advertisement amount is big with respect to the product being advertised.

10. Using the two tables below, what is $(g \circ f)(-2)$?

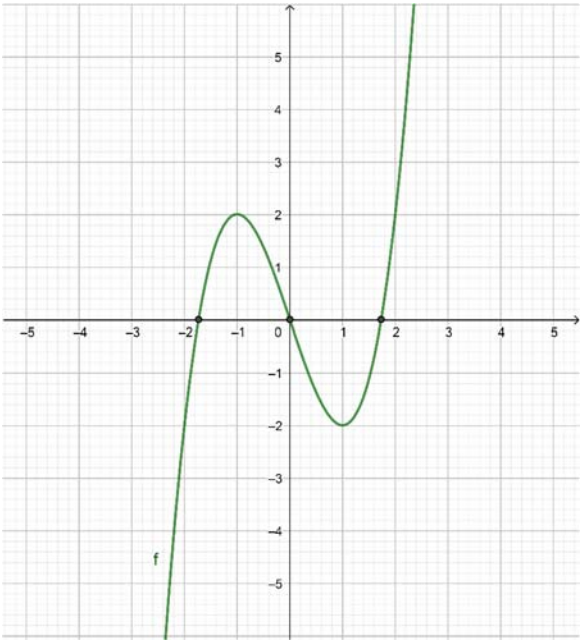
x	$f(x)$
-2	-1
4	-2
5	4

x	$g(x)$
5	4
-1	2
3	6

A. 3
B. 5
C. -2
D. -1
E. 2

11. The value, in US dollars, of a car is given by $V = \beta e^{-\alpha t}$ where t represents the age of the car in years and β and α are real numbers. The initial price of the car was \$8000, and its price after one year becomes \$6000. Then the value of α is:

A. $\ln\left(\frac{1}{4}\right)$
B. $\ln\left(\frac{1}{3}\right)$
C. $\ln\left(\frac{2}{3}\right)$
D. $\ln\left(\frac{3}{4}\right)$
E. $\ln\left(\frac{4}{3}\right)$



12. The figure above represents a function f that cuts the x-axis at three points of abscissas $-\sqrt{3}, 0$ and $\sqrt{3}$.

Consider the following intervals: $I =]-2, -\sqrt{3}[$, $K =]-1,0[$, $L =]0,1[$, $M =]1, \sqrt{3}[$ and $N =]\sqrt{3}, 2[$.

In what interval is the function f considered to be positive and decreasing function?

A. I
B. K
C. L
D. M
E. N

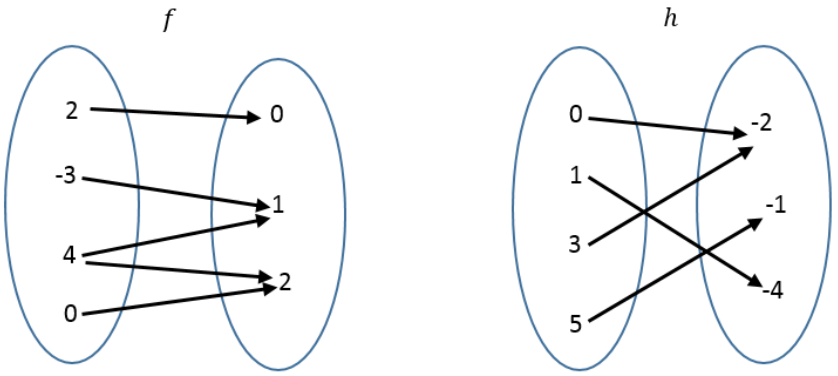
13. Consider $P(x) = ax^2 + bx + c$ (a $\neq$ 0), a polynomial of second degree in which $P(0) = 1$.

If $P(x+1) - P(x) = 4x$, what are the values of a, b and c?

A. $a = 2, b = 2, c = 0$
B. $a = 2, b = 2, c = 1$
C. $a = 1, b = 2, c = 1$
D. $a = -2, b = -2, c = 1$
E. $a = 2, b = -2, c = 1$

14. If m is a real parameter, what is the number of the real roots of the equation $x(x^2+1)(mx^2 - x - 2m) = 0$?

A. 1
B. 2
C. 3
D. 0
E. 5



15. Which of the two relations above is a function and why?

A. f because input 4 has two images
B. f because each input has exactly one output
C. h because input -2 has two pre-images
D. f or h because each relation is a function
E. h because each input has only one image

16. Consider the functions below:

$$f(x) = 2x^3 - 4x + 1$$
$$g(x) = 3x^{0.5} + 3x^{-2} + 3$$
$$h(x) = -\sqrt{3}x^2 + 3x$$
$$k(x) = \frac{1}{3}x^2 + \frac{2}{x}$$

Which of these functions is a polynomial function?

A. f and g
B. h and g
C. f and h
D. f, h and g
E. f, g, h and k

17. In the xy −plane the function f defined by $f(x) = x^3 + 4x - 5$ admits a graph (C). A computer program generated 5 graphs of which one is correct. The graphs are characterized by:

Graph 1	It admits 3 vertices.
Graph 2	It admits 2 maxima and 1 minimum.
Graph 3	It is strictly decreasing.
Graph 4	It is strictly increasing.
Graph 5	It is always positive.

Which graph is correct?

A. Graph 1
B. Graph 2
C. Graph 3
D. Graph 4
E. Graph 5

18. If $f(x) = \sqrt[3]{x^3 + 3} + 3$ and $f^{-1}(x) = \sqrt[3]{-3}$, what is the value of x?

A. -3
B. -2
C. 0
D. 3
E. 2

19. If $x + \frac{1}{x} = 2$, what is the value of $x^2 + \frac{1}{x^2}$?

A. 4
B. 8
C. 2
D. 0.5
E. 16

20. If $\log_2 16 = x + y$ and $\log_x 4 = 2$, then what is $\log_2 y$?

A. 0
B. 1
C. 2
D. 3
E. 4

21. What is the domain of definition of $arcsin(3 - 4x)$?

A. $\frac{-1}{2} \leq x \leq \frac{1}{2}$
B. $\frac{-1}{2} \leq x \leq 1$
C. $\frac{1}{2} \leq x \leq 1$
D. $-1 \leq x \leq \frac{-1}{2}$
E. $-1 \leq x \leq 1$

22. What is the parity and periodicity of function f defined by $f(x) = 2\sin\left(\frac{x}{2}\right)$?

A. even and period 4π
B. odd and period 4π
C. even and period π
D. odd and period π
E. odd and period 2π

23. $ln(x^2 - 2x + 2) > 0$ is verified in:

A. $]-\infty, +\infty[$
B. $]0, +\infty[$
C. $]0,1]$
D. $\left]-\infty, \frac{-1}{2}\right[\cup]0, +\infty[$
E. $]-\infty, 1[\cup]1, +\infty[$

$$f(x) = \begin{matrix} -8x - m & for\ x \leq 2 \\ 10x - 3 & for\ x > 2 \end{matrix}$$

24. For what value of m, if any, is the function f above continuous at $x = 2$?

A. 33
B. 17
C. 23
D. -33
E. Doesn't exist

25. If $x^2 = -2y - 5$ and $z = -8y^3$, what is x in terms of z?

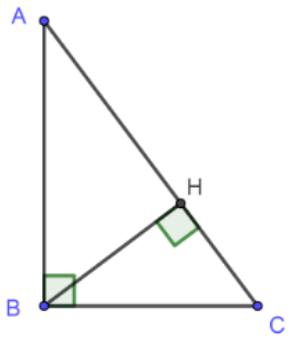
A. $x = \sqrt[3]{z} - 5$

B. $x = \pm\sqrt{\sqrt[3]{z} - 5}$

C. $x = \pm\left(\sqrt[3]{z} - 5\right)$

D. $x = \pm\sqrt[6]{z} - \sqrt{5}$

E. $x = \left(\sqrt[3]{z} - 5\right)^2$



26. Triangle ABC is right angled at B such that $B\hat{A}C = \theta$ and H is the foot of the altitude drawn from B to $[AC]$. If $AC = 4$ and $\theta = 15°$ then $BH =$

(The figure is not drawn to scale)

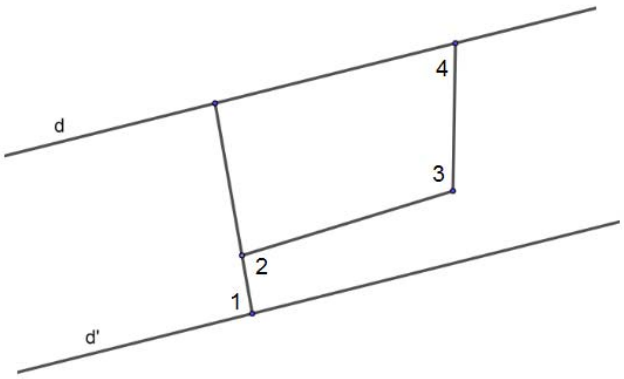
A. $\frac{1}{3}$

B. $\frac{1}{2}$

C. 1

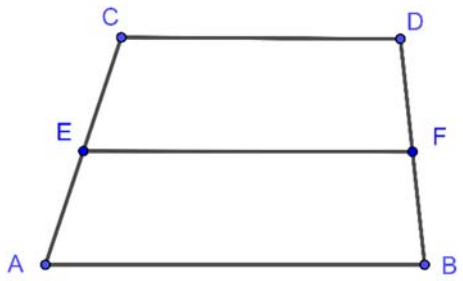
D. $\frac{2}{3}$

E. 2



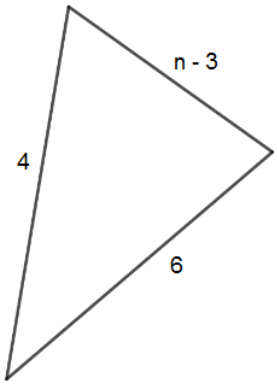
27. In the figure above, d and d′ are parallel lines, $\angle 1 = 93°, \angle 3 = 107°$ and $\angle 2 = 97°$. What is the measure of angle 4? (The figure is not drawn to scale)

A. $77°$
B. $63°$
C. $73°$
D. $83°$
E. $67°$



28. In the trapezoid above, $CD = y^2 + 1$, $AB = 3y - 1$ and $EF = 2$. If E and F are the midpoints of $\overline{AC}$ and $\overline{BD}$, what is the value of y? (The figure is not drawn to scale)

A. -4
B. 1
C. 4
D. 1 or -4
E. 1 or 4

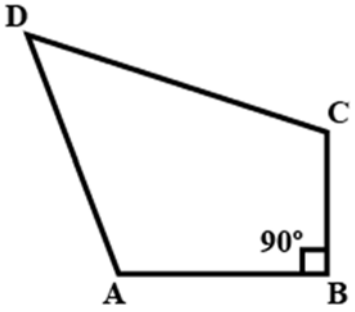


29. Which of the following numbers 2, 4, 8, 10 and 14 cannot be a value of n in the triangle above? (The figure is not drawn to scale)

A. 14
B. 2 and 4
C. 2 and 14
D. 2, 4 and 14
E. 8, 10 and 14

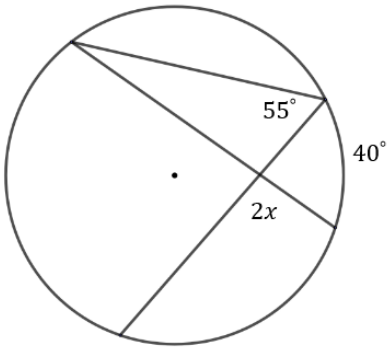
30. In the xy −plane, what quadrant(s) could point A lie in, if its x-coordinate and y-coordinate have opposite signs?

A. Quadrant I
B. Quadrant II
C. Quadrants II and III
D. Quadrants II and IV
E. Quadrants III and IV



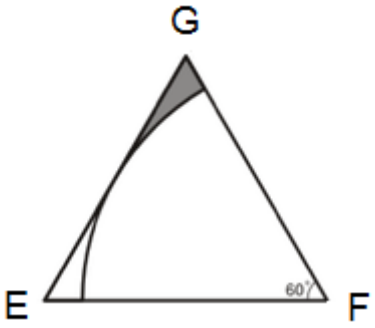
31. In the quadrilateral ABCD above, $\angle C - \angle D = 60^{\circ}$ and $\angle A - \angle C - \angle D = 10^{\circ}$. Find $\angle A$, $\angle C$ and $\angle D$. (The figure is not drawn to scale)

A. $\angle A = 140^{\circ}$; $\angle C = 95^{\circ}$; $\angle D = 35^{\circ}$
B. $\angle A = 120^{\circ}$; $\angle C = 85^{\circ}$; $\angle D = 25^{\circ}$
C. $\angle A = 95^{\circ}$; $\angle C = 140^{\circ}$; $\angle D = 80^{\circ}$
D. $\angle A = 120^{\circ}$; $\angle C = 115^{\circ}$; $\angle D = 35^{\circ}$
E. $\angle A = 140^{\circ}$; $\angle C = 35^{\circ}$; $\angle D = 95^{\circ}$



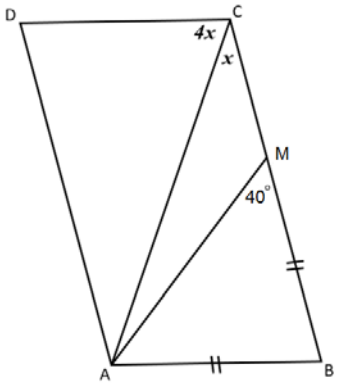
32. In the figure above, what is the value of $\frac{x}{2}$?

A. 50°
B. 66.25°
C. 105°
D. 52.5°
E. 26.25°



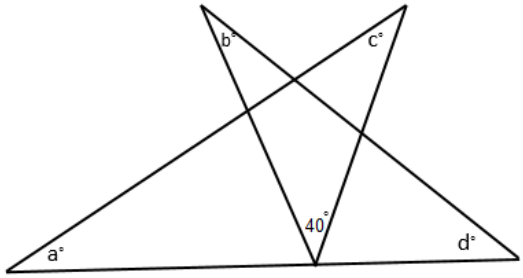
33. A circular region centered at F is inscribed in equilateral triangle EFG as shown above. If the area of the triangle EFG is $12\sqrt{3}$, what is the approximate area of the shaded region?

A. 0.97
B. 18.85
C. 6.93
D. 1.94
E. 2.17



34. In the figure above, M is a point on side BC of parallelogram $ABCD$. What is the measure of $\angle CAM$? (The figure is not drawn to scale)

A. 16°
B. 24°
C. 64°
D. 104°
E. 20°



35. What is the measure of a + b + c + d in the figure shown above? (The figure is not drawn to scale)

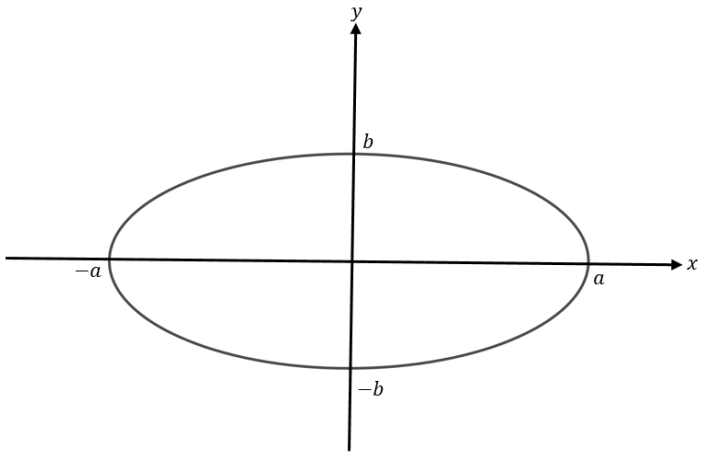
A. 140°
B. 120°
C. 160°
D. 320°
E. Can't be calculated

36. Consider the two lines D and L of respective equations $2x - y - 1 = 0$ and $mx + (m-1)y = 0$ where m is real number. For what value of m, line D is parallel to line L?

A. -1
B. 2
C. $\frac{3}{2}$
D. $\frac{2}{3}$
E. -2

37. In the $xy-$ plane, (C) is a circle of center A(1, 0) and radius 2. (d) is the line of equation $x + m = 0$. For what values of m does the line (d) cut circle (C) in two distinct points?

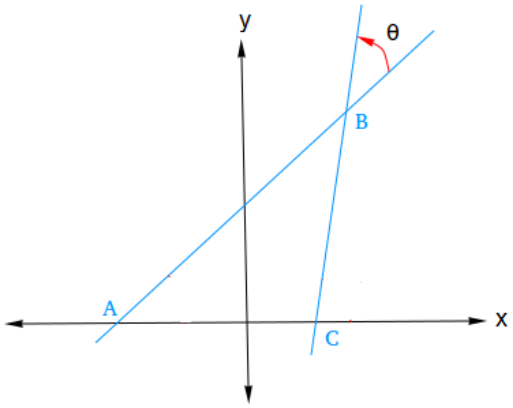
A. $m < 2$
B. $-1 < m < 3$
C. $-3 < m < 1$
D. $-2 \leq m \leq 2$
E. $m > 3$



38. Knowing that the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $A = \pi ab$, what is the equation of the ellipse if $a + b = 12$ and $A = 35\pi$?

A. $\frac{x^2}{25} + \frac{y^2}{49} = 1$
B. $\frac{x^2}{8^2} + \frac{y^2}{4^2} = 1$
C. $\frac{x^2}{7^2} + \frac{y^2}{5^2} = 1$
D. $\frac{x^2}{35^2} + \frac{y^2}{12^2} = 1$
E. $\frac{x^2}{7^2} - \frac{y^2}{5^2} = 1$

39. In the xy −plane, consider the points $A(-3,0)$, $B(3,5)$ and $C(2,0)$ as shown in the figure below. What is the approximate measure, in degrees, of the angle θ?



A. 39°
B. 40°
C. 62°
D. 79°
E. 28°

40. The point $K(1,-3)$ is rotated about the origin through an angle of 90° in an anti-clockwise direction. What are the coordinates of the image of K?

A. $(-1,-3)$
B. $(3,1)$
C. $(-3,-1)$
D. $(-3,1)$
E. $(1,3)$

41. What is the nature of triangle ABC if $A(2,3,1)$, $B(-1,3,0)$ and $C(2,3,-1)$?

A. Isosceles
B. Right
C. Right isosceles
D. Equilateral
E. Scalene

42. A hollow cube of internal edge 20 cm is filled with identical spherical marbles of diameter 0.6 cm each, and it is assumed that $\frac{1}{8}^{\text{th}}$ space of the cube remains unfilled. What is the number of marbles, rounded to the nearest integer, that the cube can accommodate?

A. 70,000
B. 61,894
C. 70,736
D. 7,737
E. 65,235

43. Triangle ABC is right at C. If $BC = x$, $AC = 2x$ where x is a positive real number and angle $B\hat{A}C = \theta$, then $\sin 2\theta =$

A. $\frac{1}{5}$
B. $\frac{2}{5}$
C. $\frac{3}{5}$
D. $\frac{4}{5}$
E. 1

44. Which could be a value of $\sin\theta$ if $\frac{7\pi}{4} < \theta < 2\pi$?

A. $\frac{-4}{5}$
B. -1
C. $\frac{-1}{2}$
D. $\frac{1}{4}$
E. $\frac{1}{2}$

45. If $cos\left(\frac{\pi}{2} - \theta\right) = \frac{2}{\sqrt{3}}$, then what is the value of $\frac{\sin\theta}{\csc\theta}$?

A. $\frac{2}{\sqrt{3}}$
B. 1
C. $\frac{4}{3}$
D. $\frac{\sqrt{3}}{2}$
E. $\frac{3}{4}$

46. If the mean of a normal distribution is 55 and the standard deviation is 5, then almost all of the scores are likely to fall between:

A. 40 and 70
B. 50 and 60
C. 30 and 55
D. 55 and 80
E. 60 and 90

47. A survey is done in an enterprise having 20% administrators and 80% employees. We know that 5% of the administrators and 20% of the employees speak Spanish.

A person is chosen randomly from the enterprise, what is the probability that he/she speaks Spanish?

A. 0.26
B. 0.17
C. 0.25
D. 0.08
E. 0.48

48. A box contains 24 marbles of red, green and yellow colors. If two marbles are drawn randomly and simultaneously from the box, the probability that both are red is $\frac{15}{92}$. What is the number of the non-red marbles?

A. 10
B. 16
C. 14
D. 20
E. 18

49. Which of the numbers below is not the mean, median, mode or range of the data set 5, 2, 4, 5, 10, 16, 12, 18, 15, 13?

A. 10
B. 11
C. 5
D. 8
E. 16

50. A regression analysis between sales (Y) and advertising (X) across all the branches in Egypt of a major company resulted in the following equation $Y = 40{,}000 + 6.25\,X$.

If the advertising budgets of the two branches in Giza and Alexandria differ by 200,000 EGP, then what will be the predicted difference in their sales?

A. 12.5 EGP
B. 80,000 EGP
C. 160,000 EGP
D. 250,000 EGP
E. 1,250,000 EGP